Received: *June 19, 2000*, Accepted: *August 11, 2000*

The SQCD vacuum coupled to supergravity and string theory moduli

Thomas Dent

Centre for Theoretical Physics, University of Sussex
Brighton BN1 9QH, UK
E-mail: t.e.dent@sussex.ac.uk

Abstract: We calculate the scalar potential of supersymmetric QCD (in the regime $N_f < N_c$) coupled to $\mathcal{N} = 1$ supergravity with moduli-dependent gauge kinetic function and masses. The gauge dynamics are described by the Taylor-Veneziano-Yankielowicz superpotential for composite effective fields. The potential can be expanded about the "truncated" point in the gaugino and matter condensate directions in order to find corrections to the globally supersymmetric minimum. The results are relevant to the phenomenology of supersymmetry-breaking in string-inspired supergravity models, and also to recent work on domain walls in SQCD.

Keywords: Supersymmetry Breaking, Supergravity Models, Supersymmetric Effective Theories.

JHEP08(2000)013

Contents

1. Introduction

Supersymmetric gauge theory with matter representations plays a central role in many models of supersymmetry-breaking relevant to string phenomenology. In $\mathcal{N}=1$ supergravity models based on perturbative string constructions, the gauge dynamics in a "hidden sector" [2] can induce a hierarchically small vacuum expectation value (v.e.v.) for the gaugino bilinear operator $\mathrm{Tr}\,\lambda^\alpha\lambda_\alpha$ (the "gaugino condensate"). If there are hidden matter multiplets [3, 4], there may also be non-zero v.e.v.'s for the squark bilinears $\bar{\tilde{q}}_{ai}\tilde{q}^a_j$ where a is the gauge group representation index and i,j are flavour indices. When coupled to supergravity the gaugino condensate is a source of supersymmetry-breaking [1] which can in principle explain the ratio of the electroweak scale, and the notional masses of supersymmetric partners, to the mass scale of string theory. The scale of SUSY-breaking can be parameterized by the gravitino mass, $m_{3/2}$, which is proportional to $\langle\mathrm{Tr}\,\lambda^\alpha\lambda_\alpha\rangle/M_P^2$ in the case of a single confining group. By using the different gauge groups and matter contents which appear in string constructions it is possible to reach phenomenologically reasonable values of $m_{3/2}$ (of order $1\,\mathrm{TeV}$) [5].

It has usually been assumed that the structure of the gauge theory vacuum is well described by *globally* supersymmetric QCD, since the strong-coupling scale is well below M_P. In the limit of global supersymmetry $M_P \to \infty$ the condensates do not break supersymmetry [6, 7, 8]. The condensate values may be found by instanton calculations [9] or by constructing a supersymmetric effective action [6, 7] in terms of gauge invariant composite superfields $U \propto \mathrm{Tr}\,W^\alpha W_\alpha$ and $V_{ij} \propto \bar{Q}_{ai}Q^a_j$, where W^α is the supersymmetric gauge field strength and $Q, \bar{Q}$ are the quark and antiquark chiral superfields respectively. The supersymmetric zero-energy vacuum satisfies the conditions $F_U = -\partial W^*_{\rm np}/\partial U^* = 0$, $F_V = -\partial W^*_{\rm np}/\partial V^*_{ij} = 0$, where $W_{\rm np}$ is the non-perturbative superpotential generated by the gauge dynamics. The gaugino condensate value is then determined (up to a discrete symmetry) in terms of the renormalisation group invariant scale Λ and the squark mass matrix M_{ij}. In string-inspired supergravity models the scale Λ and the squark masses are functions of the dilaton and moduli scalars of the underlying theory. Hence the supersymmetry-breaking has a non-trivial dependence on the "flat directions" of string theory and it becomes possible to stabilize the moduli [10, 11] and (under certain assumptions!) the dilaton [12], while inducing supersymmetry-breaking of the right magnitude [5, 13].

Recently, SQCD has also been used in the construction of (supersymmetric) domain walls [14], motivated by cosmological issues or by the "brane world" scenario in which the observable matter fields are confined to a region localized in one or more directions within a higher-dimensional spacetime. The discrete set of degenerate vacua that is required is naturally realised in SU(N) SQCD, since the vacuum with non-zero gluino and matter condensates transforms under a non-anomalous $\mathbb{Z}_N$ symmetry.[1] When supersymmetry is broken by an explicit gluino mass term [15], the degeneracy is also broken, which raises doubts as to the stability of domain walls when gravitational corrections to the scalar potential are included. A non-zero v.e.v. for $\mathrm{Tr}\,\lambda^\alpha\lambda_\alpha$ will in general break (local) SUSY and give the gluinos, including hidden sector gauginos, a mass. We will find, however, that the vacuum degeneracy is *unbroken* when the source of SUSY-breaking is the gaugino condensate itself.

In this paper we attempt to find the corrections to the vacuum structure of SQCD due to its coupling to $\mathcal{N} = 1$ supergravity, including the dilaton and an overall modulus denoted by chiral superfields S and T respectively. The starting point will be the non-perturbative superpotential of [7] and the resulting zero-energy vacuum of the globally supersymmetric theory. It has already been shown in the case of a SYM hidden sector *without* matter, that the resulting value of the gaugino condensate is identical to that at the minimum of the scalar potential $\mathcal{V}(U, S, T)$ in $\mathcal{N} = 1$ supergravity with dilaton and modulus fields, when terms in U of mass dimension higher than 4 (i.e. suppressed by powers of M_P) are discarded [16, 4]. We will show that this also holds in the case of SQCD: on discarding the terms in the supergravity potential with mass dimension > 4 in the condensate fields (note that we treat S and T as being dimensionless), we recover the global SQCD vacuum. The size of deviations from the "truncated approximation" [3] can be found by minimising the scalar potential in supergravity as a function of U and V_{ij}, including higher-order terms. A similar approach was taken in the case of pure SYM coupled to supergravity in [11, 17, 18].

[1] This is the subgroup of the anomalous $\mathrm{U}(1)_R$ that survives breaking by instantons: in general there will be a $\mathbb{Z}_{c(G)}$ degeneracy.

2. Effective action for the gaugino and squark condensates

We follow the approach of Burgess et al. [19] in which the gaugino condensate is described by the *classical* field $U \equiv \hat{U}/S_0^3$, where $\hat{U} = \langle \mathrm{Tr}\, W^\alpha W_\alpha \rangle$ and the chiral compensator superfield S_0 is introduced to simplify the formulation of supergravity coupled to matter [20] using the superconformal tensor calculus.[2] The gaugino bilinear $\langle \mathrm{Tr}\, \lambda^\alpha \lambda_\alpha \rangle$ corresponds to the lowest component of $\hat{U}$ ($\theta = \bar{\theta} = 0$). Similarly the squark condensate is the lowest component (mass dimension 2) of a composite superfield $V_{ij} = \langle \bar{Q}_{ai} Q_j^a \rangle$ where Q_i, $\bar{Q}_j$ are N_f flavours of left chiral superfields in the representation R and its complex conjugate $\bar{R}$ respectively. The v.e.v.'s of the gaugino and squark bilinears are then given by the scalar components of U and V_{ij} at the stationary point of the effective action $\Gamma(U, V, S, T)$ [19].

The effective action results from the standard $\mathcal{N} = 1$ supergravity formula with the Taylor-Veneziano-Yankielowicz (TVY) non-perturbative superpotential in terms of U and V, and an appropriate Kähler potential K. The TVY superpotential, suitably amended for the case of a non-minimal gauge kinetic function and a modulus-dependent mass matrix, is

$$W(U, V_{ij}, S, T) = \frac{1}{4} f_G(S,T) U - \frac{1}{96\pi^2} U \ln \left(k U^{b+2c} (\det V_{ij})^{-3c/N_f} \right) - M_{ij}(T) V_{ij} \,. \quad (2.1)$$

Here $f_G(S,T)$ is the gauge kinetic function, equal to S at tree level, which in general depends on the modulus T through string loop threshold corrections [21], b is the one-loop beta function coefficient such that $b = -3c(G) + 2N_f T(R)$, and $c = 2N_f T(R)$; $c(G)$ is the second-order Casimir invariant for the gauge group G and $T(R)$ is the index, equal to $1/2$ in the fundamental representation. k is a constant which will be discussed shortly. It is convienient to diagonalise V_{ij} by performing unitary flavour rotations of the matter fields Q_i, $\bar{Q}_j$ so that $V_{ij} = V_i \delta_{ij}$. The superpotential can then be re-expressed as

$$W = -\frac{b}{96\pi^2} U \ln \left(k U^{1+2c/b} \prod_i V_i^{-3c/N_f b} \omega(S) h(T) \right) - M_i(T) V_i \,, \qquad (2.2)$$

where $\omega(S) \equiv e^{-24\pi^2 S/b}$ and $h(T)$ is a function which transforms under the target-space duality group $\mathrm{SL}(2, \mathbb{Z})$ [11, 22] such that the argument of the logarithm in (2.2) is invariant.

This is all we need to find the truncated approximation for the condensate values: it simply follows from setting the derivatives of W with respect to U and V to zero, giving us

$$U^{(\mathrm{tr})} = e^{24\pi^2 f_G(S,T)/b_0 - (b+2c)/b_0} k^{-b/b_0} \left(\frac{c}{32\pi^2 N_f} \right)^{3c/b_0} \prod_i M_i(T)^{-3c/N_f b_0} \qquad (2.3)$$

[2]The components of S_0 are determined by gauge-fixing the superconformal symmetries so that the Einstein term in the lagrangian is canonically normalised [19].

and

$$V_i^{(\mathrm{tr})} = \frac{c}{32\pi^2 N_f} \frac{U^{(\mathrm{tr})}}{M_i(T)}\,, \tag{2.4}$$

where $b_0 = -3c(G)$ is the beta-function coefficient for SYM without matter. We see that the constant k simply adjusts the scale of the condensates; it has the same effect as a constant threshold correction to f_G.

When the condensate values (2.3) and (2.4) are substituted back into the superpotential, the "truncated superpotential" $W_{\mathrm{np}}^{(\mathrm{tr})}(S,T) = (b_0/96\pi^2)U^{(\mathrm{tr})}$ emerges. This is the usual starting point for studying supersymmetry-breaking in string effective field theories (see for example [23]).

We take the Kähler potential for the dilaton and moduli to be

$$\tilde{K} = P(y) - 3\ln(T+T^*)\,, \tag{2.5}$$

where $y = S + S^* - (1/8\pi^2)\delta_{\mathrm{GS}}\ln(T+T^*)$. The string tree-level Kähler potential for the dilaton $-\ln(S+S^*)$ has been replaced by a real function $P(y)$ which parameterizes stringy non-perturbative dilaton dynamics [24, 12]. We take the Green-Schwarz coefficients δ_{GS} to be zero to simplify the calculations. The correct form of $P(y)$ is not known, however it is possible to constrain it by looking for a stable minimum in the potential for the dilaton and requiring $P''(y) > 0$ to obtain the right sign kinetic term.

The $\mathcal{N}=1$ supergravity effective action is invariant under target-space $\mathrm{SL}(2,\mathbb{Z})$ transformations [25, 26] if the superpotential transforms as a modular form[3] of weight -3. The modulus T transforms as

$$T \longrightarrow \frac{\alpha T - i\beta}{i\gamma T + \delta}\,, \tag{2.6}$$

where $\alpha, \beta, \gamma, \delta$ are integers satisfying $\alpha\delta - \beta\gamma = 1$. Then U must transform as

$$U \longrightarrow \zeta(i\gamma T + \delta)^{-3} U\,, \tag{2.7}$$

where ζ is a phase factor which depends on α, β, γ and δ. Since the gauge fields are inert under target-space modular transformations this is achieved by S_0 having a non-trivial transformation property. The transformation of the squark condensate V is determined by that of the $Q, \bar{Q}$ fields, which we take to have (flavour-independent) modular weights $n_Q, n_{\bar{Q}}$: then $V_i \to \zeta_V (i\gamma T + \delta)^{n_Q + n_{\bar{Q}}} V_i$. The function $h(T)$ is determined by string threshold corrections (up to multiplication by a modular invariant function of T) as $h(T) = \eta(T)^{6b'/b}$, where $\eta(T)$ is the Dedekind eta-function and the coefficient b' is [28] $b' = 3(-c(G) + \sum_Q T(R_Q)(1+2n_Q))$ for a gauge group with twisted matter representations R_Q of modular weight n_Q.

[3] For a discussion of modular forms see [27] or the appendix of [23].

The complete Kähler potential is then taken to be

$$K(U,V_i,S,T) = \tilde{K} - 3\ln\left(1 - Ae^{\tilde{K}/3}(UU^*)^{1/3}\right) + B(T+T^*)^{\bar{n}}(V_iV_i^*)^{1/2}\,, \qquad (2.8)$$

where A and B are constants and $\bar{n} \equiv (n_Q + n_{\bar{Q}})/2$. It was shown in [19] that this expression for the Kähler potential of U has the correct dependence on S and T; the V-dependent part is fixed by the requirements to respect $U(N_f)$ flavour symmetry and modular invariance. The Kähler potential for the composite fields U and V is only determined up to constant factors, and may receive higher-order corrections (which, however, will be negligible when the field values are small). The constants A, B cannot at present be computed, due to our incomplete knowledge of supersymmetric gauge dynamics, but they are expected to be of order unity. Since the scalar potential in supergravity is a function of K as well as the superpotential W these constants will also appear in our results. In an earlier paper [18] the constant A was absorbed by rescaling the gaugino condensate, however since physical SUSY-breaking quantities are expressed in terms of the *un-rescaled* gaugino condensate (and, in general, the normalisation of fields is fixed by the coefficients with which they enter into W) this will not be done here.

The part of $\Gamma(U,V_i,S,T)$ relevant to finding the value of the condensate is the scalar potential, which is given as usual by[4]

$$\mathcal{V} = e^K\left((W_I^* + K_IW^*)(K^{-1})^I_J(W^J + K^JW) - 3|W|^2\right),$$

where I and J range over the scalar components of U, V_i, S and T, $X^I \equiv \partial X/\partial\phi_I$ and $X_J \equiv \partial X/\partial\phi^{J*}$ for X any function of the scalars and their complex conjugates, and K^{-1} is defined by $(K^{-1})^I_J K^J_K = \delta^I_K$. The details of the inverse Kähler metric components are relegated to the appendix.

As emphasized in [29], $\Gamma(U,V_i,S,T)$ is not an effective lagrangian in the sense of describing the light degrees of freedom only, (for one thing it depends on the heavy fields U and V), rather it is the generating functional of "two-particle irreducible" correlation functions for the composite fields $\mathrm{Tr}\,W^2$ and $\mathrm{Tr}\,\bar{Q}_iQ_j$ (see also [19]). The kinetic terms in Γ must be understood as the first terms in a derivative expansion; it may well be that there are large higher-derivative corrections, so the effective action cannot be reliably used to determine particle interaction vertices. However the scalar potential, i.e. $\Gamma(U,V_i,S,T)$ for constant field configurations, does not get these corrections.

After some calculation, we find the scalar potential in closed form, which for convenience is expressed in terms of the rescaled quantities $z = e^{\tilde{K}/6}U^{1/3}$, $\Pi_i = (T+T^*)^{\bar{n}}V_i$, $\|\Pi\| = (\Pi_i\Pi_i^*)^{1/2}$:

$$\mathcal{V} = \frac{e^{B\|\Pi\|}}{(1-A|z|^2)^3}\left\{\mathcal{V}_0 + \mathcal{V}_1\right\} \qquad (2.9)$$

[4] We work in reduced Planck units with $\kappa^{-1} = 1/\sqrt{8\pi G} = 1$.

with

$$\begin{aligned}\mathcal{V}_0 =& \left(\frac{b}{96\pi^2}\right)^2 \frac{3|z|^4}{A}(1-A|z|^2)\left|1+\frac{2c}{b}+\mathcal{LOG}\right|^2 + \\ &+ \frac{2}{B\|\Pi\|}\left(\delta_{ij}\|\Pi\|^2+\Pi_i^*\Pi_j\right)\left(\frac{c}{32\pi^2 N_f} z^{*3}\Pi_i^{*-1} - e^{\tilde{K}/2}(T+T^*)^{-\bar{n}}M_i(T^*)\right)\times \\ &\times\left(\frac{c}{32\pi^2 N_f} z^{3}\Pi_j^{-1} - e^{\tilde{K}/2}(T+T^*)^{-\bar{n}}M_j(T)\right),\end{aligned}\tag{2.10}$$

$$\begin{aligned}\mathcal{V}_1 =& \frac{P'(y)^2}{P''(y)}(1-A|z|^2)\mathcal{V}_S + \frac{1-A|z|^2}{3\left(1-\frac{\bar{n}}{3}(1-A|z|^2)B\|\Pi\|\right)}\mathcal{V}_T + \\ &+ \left(\frac{b}{96\pi^2}\right)^2|z|^6\left(-3\left(1+\frac{2c}{b}\right)^2-\frac{6c}{b}(\mathcal{LOG}+\text{c.c.})+B\|\Pi\|\cdot|\mathcal{LOG}|^2\right)+ \\ &+\frac{b}{96\pi^2}(2+B\|\Pi\|)\left(e^{\tilde{K}/2}(T+T^*)^{-\bar{n}}z^3\mathcal{LOG}\cdot M_i(T^*)\Pi_i^*+\text{c.c.}\right)+ \\ &+\frac{3b}{96\pi^2}\left(1-A|z|^2\left(1+\frac{2c}{b}\right)\right)\left(e^{\tilde{K}/2}(T+T^*)^{-\bar{n}}z^3M_i(T^*)\Pi_i^*+\text{c.c.}\right)+ \\ &+e^{\tilde{K}}(T+T^*)^{-2\bar{n}}\left(1+3A|z|^2+B\|\Pi\|\right)|M_i(T)\Pi_i|^2,\end{aligned}\tag{2.11}$$

where

$$\mathcal{LOG} = \ln\left(kU^{1+2c/b}\prod_i V_i^{-3c/N_f b}\omega(S)h(T)\right),\tag{2.12}$$

$$\begin{aligned}\mathcal{V}_S =& \left|\frac{b}{96\pi^2}z^3\left(\frac{1}{P'(y)}\frac{\omega'(S)}{\omega(S)}-\left(1+\frac{2c}{b}\right)\right)+e^{\tilde{K}/2}(T+T^*)^{-\bar{n}}M_i(T)\Pi_i\right|^2, \\ \mathcal{V}_T =& \left|\frac{b}{96\pi^2}z^3\left((T+T^*)\frac{h'(T)}{h(T)}+3\left(1+\frac{2c}{b}(1+\bar{n})\right)\right)+\right. \\ &\left.+e^{\tilde{K}/2}(T+T^*)^{-\bar{n}}\left((T+T^*)\frac{M_i'(T)}{M_i(T)}-(3+2\bar{n})\right)M_i(T)\Pi_i\right|^2.\end{aligned}\tag{2.13}$$

In this somewhat unwieldy expression, $\mathcal{V}_0$ includes the terms of mass dimension 4 [glossing over factors of $(1-A|z|^2)$], which survive in the global supersymmetry limit $M_P\to\infty$, $z\to 0$, $\Pi\to 0$, while $\mathcal{V}_1$ includes all terms of higher order in z and Π. Note also the "modular covariant" expressions

$$\hat{G}^T \equiv (T+T^*)\frac{h'(T)}{h(T)}+3\left(1+\frac{2c}{b}(1+\bar{n})\right),\qquad \hat{G}^{M_i}\equiv(T+T^*)\frac{M_i'(T)}{M_i(T)}-(3+2\bar{n})$$

appearing in $\mathcal{V}_T$, which transform into themselves up to a phase under $\mathrm{SL}(2,\mathbb{Z})$. For future convenience we also define

$$\hat{G}^S \equiv \frac{1}{P'(y)}\frac{\omega'(S)}{\omega(S)}-\left(1+\frac{2c}{b}\right).$$

As expected, $\mathcal{V}_0$ has a zero-value minimum in the condensate directions[5] at $z = z^{(\mathrm{tr})} \equiv e^{\tilde{K}/6} U^{(\mathrm{tr})1/3}$ and $\Pi_i = \Pi_i^{(\mathrm{tr})} \equiv (T+T^*)^{\bar{n}} V_i^{(\mathrm{tr})}$. The "truncated superpotential" result for the scalar potential is then equivalent to substituting these condensate values back into the full potential, which is then proportional to $\mathcal{V}_1$, and discarding terms of dimension greater than 6 in z and Π_i. It is a non-trivial check on the result (2.11) to carry out this substitution and verify that the scalar potential then coincides with previous results [22, 4]. In fact we obtain

$$\begin{aligned}\mathcal{V}^{(\mathrm{tr})} &\equiv \mathcal{V}(S,T)_{|z=z^{(\mathrm{tr})},\Pi=\Pi^{(\mathrm{tr})}} = \\ &= \left(\frac{b_0}{96\pi^2}\right)^2 |z^{(\mathrm{tr})}|^6 \left\{ \frac{1}{P''(y)} \left| \frac{\omega_0'(S)}{\omega_0(S)} - P'(y) \right|^2 + \right. \\ &\qquad \left. + \frac{1}{3} \left| (T+T^*) \frac{h_0'(T)}{h_0(T)} + 3 \right|^2 - 3 \right\} + \mathcal{O}\left(z^{(\mathrm{tr})8}\right),\end{aligned}$$

where $\omega_0(S) = e^{-24\pi^2 S/b_0}$, $h_0(T) = \eta(T)^{6b'/b_0} \prod_i M_i(T)^{3c/N_f b_0}$. Note that the unknown constants A and B drop out of this expression. This occurs because the truncated approximation for the condensates is implemented via the superpotential W_{np} alone, and does not (at the given order of approximation in z/M_P) affect the Kähler potential for S and T.

3. Beyond the truncated approximation

The truncated approximation relies on the assumption that the condensate values at the minimum of $\mathcal{V}_0$ is not much changed when the gravitationally-suppressed terms in $\mathcal{V}_1$ are added. This is likely to be true when the condensate values z, Π are small, which is also the phenomenologically interesting regime for most applications. We are able to test this assumption explicitly by expanding about the truncated values to find the shift in the minimum. As a simplifying assumption the masses $M_i(T)$ are set equal to $M(T)$ and the squark condensates Π_i equal to Π. We define the fractional changes in the condensate values

$$\Delta z = \frac{z}{z^{(\mathrm{tr})}} - 1\,, \qquad \Delta\Pi = \frac{\Pi}{\Pi^{(\mathrm{tr})}} - 1$$

and their real and imaginary parts $\mathrm{Re}\,\Delta z, \mathrm{Im}\,\Delta z, \mathrm{Re}\,\Delta\Pi, \mathrm{Im}\,\Delta\Pi$. We then expand the scalar potential (2.9) up to terms of mass dimension 6 in the fields (in fact, we would expect higher-order corrections to the Kähler potential for U and V_i to become significant beyond this order) and up to quadratic order in Δz and $\Delta\Pi$. The result takes the form

$$\mathcal{V} = \mathcal{V}^{(\mathrm{tr})} + \vec{\mathcal{V}}_L^T \cdot \vec{\Delta} + \vec{\Delta}^T \underline{\underline{\mathcal{V}_Q}} \vec{\Delta}\,, \tag{3.1}$$

[5] Note that the *principal value* of the logarithm in (2.12) must be taken: since the argument is real *in vacuo* this does not cause an ambiguity.

where $\vec{\Delta}$ is the column-vector with entries $\mathrm{Re}\,\Delta z, \mathrm{Im}\,\Delta z, \mathrm{Re}\,\Delta\Pi, \mathrm{Im}\,\Delta\Pi$, and $\vec{\mathcal{V}}_L$ and $\underline{\underline{\mathcal{V}_Q}}$ are a vector and symmetric matrix of coefficients respectively. The displacements $\vec{\Delta}_{\mathrm{min}}$ at the minimum of $\mathcal{V}$ are given by

$$\vec{\Delta}_{\mathrm{min}} = -\frac{1}{2}\underline{\underline{\mathcal{V}_Q}}^{-1}\cdot\vec{\mathcal{V}}_L \tag{3.2}$$

and the change in the potential from the truncated value is

$$\mathcal{V}_{\mathrm{min}} - \mathcal{V}^{(\mathrm{tr})} = -\frac{1}{4}\vec{\mathcal{V}}_L^T\underline{\underline{\mathcal{V}_Q}}^{-1}\vec{\mathcal{V}}_L\,. \tag{3.3}$$

The coefficients in $\underline{\underline{\mathcal{V}_Q}}$ are of dimension four and six in the fields (and in the modulus-dependent mass $M(T)$) while those in $\vec{\mathcal{V}}_L$ are dimension six only (since they originate from $\mathcal{V}_1$). Since we are evaluating $\vec{\Delta}_{\mathrm{min}}$ and $\mathcal{V}_{\mathrm{min}}$ to leading order in $z^{(\mathrm{tr})}$ we can discard dimension six terms in $\underline{\underline{\mathcal{V}_Q}}$, provided that the matrix does not become singular. Then we have

$$\begin{aligned}\mathcal{V}_L^T\cdot\vec{\Delta} = \frac{\left|z^{(\mathrm{tr})}\right|^6}{(32\pi^2)^2}\Bigg[&\mathrm{Re}\,\Delta z\left(\frac{P'(y)^2}{P''(y)}\left(\frac{2b^2}{3}(\hat{G}^S)^2+2bc\hat{G}^S\right)+\frac{2b^2}{9}\left|\hat{G}^T\right|^2+\right.\\ &\left.+\frac{2bc}{3}\,\mathrm{Re}(\hat{G}^T\hat{G}^{M*})-2b\cdot b_0\right)+\mathrm{Im}\,\Delta z\cdot-\frac{2bc}{3}\,\mathrm{Im}(\hat{G}^T\hat{G}^{M*})+\\ &+\mathrm{Re}\,\Delta\Pi\left(\frac{P'(y)^2}{P''(y)}\left(\frac{2bc}{3}\hat{G}^S+2c^2\right)+\frac{2c^2}{3}|\hat{G}^M|^2+\right.\\ &\left.+\frac{2bc}{9}\,\mathrm{Re}(\hat{G}^T\hat{G}^{M*})+\frac{2b_0c}{3}\right)+\mathrm{Im}\,\Delta\Pi\cdot\frac{2bc}{9}\,\mathrm{Im}(\hat{G}^T\hat{G}^{M*})\Bigg]\end{aligned} \tag{3.4}$$

and

$$\begin{aligned}&\vec{\Delta}^T\underline{\underline{\mathcal{V}_Q}}\vec{\Delta} =\\ &=\frac{|z^{(\mathrm{tr})}|^4}{(32\pi^2)^2}\Bigg[\frac{3}{A}\Big((b+2c)^2\left((\mathrm{Re}\,\Delta z)^2+(\mathrm{Im}\,\Delta z)^2\right)-\\ &\qquad -2c(b+2c)(\mathrm{Re}\,\Delta z\,\mathrm{Re}\,\Delta\Pi+\mathrm{Im}\,\Delta z\,\mathrm{Im}\,\Delta\Pi)+\\ &\qquad +c^2\left((\mathrm{Re}\,\Delta\Pi)^2+(\mathrm{Im}\,\Delta\Pi)^2\right)\Big)+\frac{4(32\pi^2)cN_f^{1/2}e^{\tilde{K}/2}|M(T)|}{B|z^{(\mathrm{tr})}|(T+T^*)^{\tilde{n}}}\times\\ &\qquad\times\left(9\left((\mathrm{Re}\,\Delta z)^2+(\mathrm{Im}\,\Delta z)^2\right)+(\mathrm{Re}\,\Delta\Pi)^2+(\mathrm{Im}\,\Delta\Pi)^2\right)+\mathcal{O}(|z^{(\mathrm{tr})}|^6)\Bigg]\,.\end{aligned} \tag{3.5}$$

Note that the quadratic terms originating from $\mathcal{V}_0$ fall into two parts, reflecting their origins in $|F_U|^2$ and $|F_V|^2$, and there is an explicit dependence on the mass $M(T)$. Without the terms proportional to M, the matrix $\underline{\underline{\mathcal{V}_Q}}$ would be singular, reflecting the runaway instability in *massless* globally supersymmetric QCD. With the mass terms $\underline{\underline{\mathcal{V}_Q}}$ can easily be inverted and it is a matter of algebra to find $\vec{\Delta}_{\mathrm{min}}$ and $\mathcal{V}_{\mathrm{min}}$.

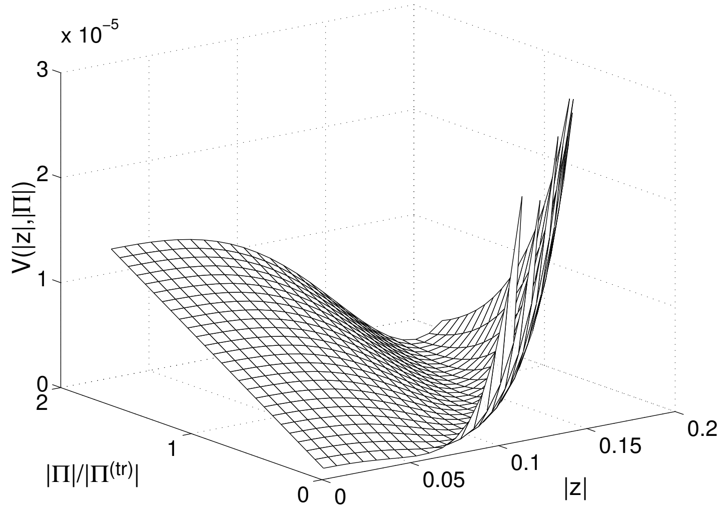

Figure 1: Full scalar potential $\mathcal{V}(|z|,|\Pi|)$ ($|z^{(\mathrm{tr})}| = 0.15, |M| = 0.07$, other parameters as specified in the text).

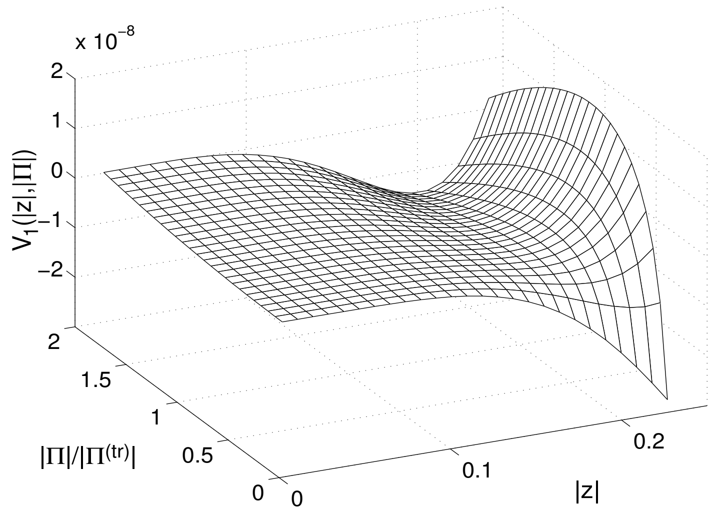

Figure 2: Supergravity corrections $\mathcal{V}_1(|z|,|\Pi|)$.

If the functions G^T and G^M have the same phase then the deviations Δz and $\Delta\Pi$ are real. In this case we can get an intuitive idea by plotting the dependence of the full potential (2.9) on the absolute values of the condensates $|z|, |\Pi|$ (given that their phases are fixed to the truncated values). Figure 1 shows the scalar potential $\mathcal{V}(|z|,|\Pi|)$ for $b_0 = -12$, $c = 2$, $|z^{(\mathrm{tr})}| = 0.15$ and $M = 0.07$, with $\hat{G}^S, \hat{G}^T, \hat{G}^M$ set to the (somewhat arbitrary) values of $0.5, 3.0, 0.0$ respectively. Figure 2 shows the gravitationally-suppressed corrections $\mathcal{V}_1(|z|,|\Pi|)$ for the same values of parameters. The structure of these terms is much richer than in the case without matter multiplets: however along the curve specified by $\partial W/\partial\Pi = 0$, the form of the scalar potential reduces to the case of pure SYM (see e.g. [22]) (figure 3).

JHEP08(2000)013

There remains the question of the fate of the N degenerate vacua in SU(N) SQCD when supersymmetry is broken. Under the $\mathbb{Z}_N$ symmetry the condensates transform as $U \to e^{2i\pi/N}U, V_i \to e^{2i\pi/N}V_i$. It is known [30, 29] that the logarithmic term in the TVY effective action does not respect the symmetry, since the branch cut is crossed. There are in practice various ways of restoring the symmetry for the $\mathbb{Z}_N$ transformed vacua, including giving the squark condensates extra 2π phases (see the first reference of [14]) such that the argument of the logarithm is invariant, adding extra fields [29], or shifting the theta-angle by 2π (equivalent to taking $S \to S + i/8\pi^2$) which will exactly cancel the effect of an axial $\mathbb{Z}_N$ transformation. Taking such "fixes" into account, the scalar potential (2.9) is invariant under the $\mathbb{Z}_N$ symmetry by inspection, since z^3 and Π_i get the same phase. While this is a reassuring result for the construction of domain walls, we must ask why it appears to contradict the reasoning that any gaugino mass must break the vacuum degeneracy. We certainly expect that the (*hidden sector*) gauginos will get a small (in general complex) SUSY-breaking mass in the case we have studied. However, since the source of the mass is the gaugino condensate itself, *the gaugino mass is not inert under the* $\mathbb{Z}_{\mathbb{N}}$ *symmetry.* In fact the mass term will be proportional to $z^{*3}\,\mathrm{Tr}\,\lambda\lambda$ (see for example the first reference of [13]), which *is* invariant.[6]

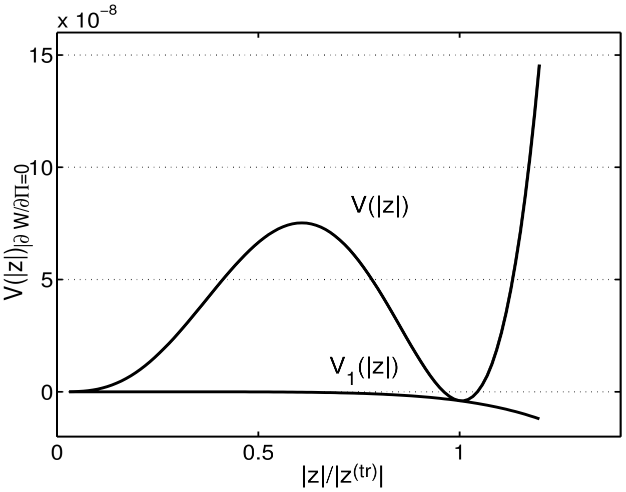

Figure 3: $\mathcal{V}(|z|)$ and $\mathcal{V}_1(|z|)$ along the curve $\partial W/\partial\Pi = 0$ for $|z|^{(\mathrm{tr})} = 0.15$ and a negative cosmological constant.

4. Conclusions

Given an explicit form of the Kähler potential for the composite superfields representing the condensates, the scalar potential in a string-inspired $\mathcal{N} = 1$ supergravity model can be calculated, which allows us to find the minimum in the condensate directions without any assumptions about the presence or absence of supersymmetry-breaking. The deviation from the "truncated" globally supersymmetric vacuum of SQCD have been found in terms of the dilaton and string moduli. The phenomenolog-

[6]To see this, it is necessary to use the formulas for the $\mathcal{N} = 1$ supergravity action in terms of W and K separately, rather than the Kähler function $G = K + \log|W|^2$, since in going from one formulation to the other the gaugino fields are redefined by a complex phase and the phase of W is eliminated [31].

ical implications of deviations from the truncated approximation were discussed in an earlier paper [18]. In the case of hidden matter, there is another possible cause for Δz, $\Delta\Pi$ to be large, namely if $\hat{G}^M$ becomes large due to singular behaviour of the mass $M(T)$ at some points in moduli space. Since $\det \underline{\underline{\mathcal{V}_Q}}$ is proportional to M, the deviations may also be large if M becomes small. However, in the limit $M \ll z$ the SQCD vacuum undergoes a phase transition and the matter fields themselves (rather than the composite operator $\mathrm{Tr}\,\bar{Q}Q$) can get v.e.v's, so our effective action will not be valid.

While the particular model from which this potential is derived, inspired by orbifolds of the heterotic string, is quite restrictive in its dependence on the dilaton and moduli, it would be relatively simple to extend the result to the case when the gauge kinetic function and mass matrix depend on several moduli, or the Kähler potential for the moduli is modified from its tree-level form (e.g. by introducing functions $P_i(T_i + T_i^*)$ where T_i is the i-th modulus). So our approach should be applicable to supergravity effective theories based on heterotic M-theory [32] or type-IIB string models [33, and references therein]. In particular we expect that the supergravity corrections will preserve the discrete symmetry of degenerate vacua in SYM and SQCD.

Acknowledgments

Thanks are due to David Bailin for supervising the work and Beatriz de Carlos for an enlightening discussion. TD is supported by PPARC studentship PPA/S/S/1997/02555.

A. Inverse Kähler metric components

The inverse Kähler metric components are as follows:

$$
\begin{aligned}
(K^{-1})_U^U &= \frac{(1-A|z|^2)|z|^4}{Ae^{\tilde{K}}}\left(3 + \frac{\bar{n}AB|z|^2(1-A|z|^2)\|\Pi\|}{1-\frac{\bar{n}}{3}B(1-A|z|^2)\|\Pi\|} + \frac{A|z|^2P'(y)^2}{P''(y)}\right), \\
(K^{-1})_{V_i}^U &= \frac{-2\bar{n}z^{*3}(1-A|z|^2)\Pi_i}{e^{\tilde{K}/2}(T+T^*)^{\bar{n}}\left(1-\frac{\bar{n}}{3}B(1-A|z|^2)\|\Pi\|\right)}, \\
(K^{-1})_S^U &= -\frac{z^{*3}(1-A|z|^2)P'}{e^{\tilde{K}/2}P''}, \qquad (K^{-1})_T^U = \frac{(T+T^*)z^{*3}(1-A|z|^2)}{e^{\tilde{K}/2}\left(1-\frac{\bar{n}}{3}B(1-A|z|^2)\|\Pi\|\right)}, \\
(K^{-1})_{V_j}^{V_i} &= \frac{2}{B(T+T^*)^{2\bar{n}}\|\Pi\|}\left(\delta_{ij}\|\Pi\|^2 + \Pi_i^*\Pi_j\frac{1-\frac{\bar{n}}{3}(1-2\bar{n})(1-A|z|^2)B\|\Pi\|}{1-\frac{\bar{n}}{3}(1-A|z|^2)B\|\Pi\|}\right), \\
(K^{-1})_S^{V_i} &= 0 = (K^{-1})_T^S, \qquad (K^{-1})_T^{V_i} = -\frac{2\bar{n}(T+T^*)(1-A|z|^2)\Pi_i^*}{3(T+T^*)^{\bar{n}}(1-\frac{\bar{n}}{3}B(1-A|z|^2)\|\Pi\|)}, \\
(K^{-1})_S^S &= \frac{1-A|z|^2}{P''(y)}, \qquad (K^{-1})_T^T = \frac{(T+T^*)^2(1-A|z|^2)}{3\left(1-\frac{\bar{n}}{3}B(1-A|z|^2)\|\Pi\|\right)},
\end{aligned}
$$

where $z = e^{\tilde{K}/6}U^{1/3}$, $\Pi_i = (T+T^*)^{\bar{n}}V_i$, $\|\Pi\| = (\Pi_i\Pi_i^*)^{1/2}$.

JHEP08(2000)013

References

[1] H.P. Nilles, *Dynamically broken supergravity and the hierarchy problem*, *Phys. Lett.* **B 115** (1982) 193;
S. Ferrara, L. Girardello and H.P. Nilles, *Breakdown of local supersymmetry through gauge fermion condensates*, *Phys. Lett.* **B 125** (1983) 457.

[2] J.P. Derendinger, L.E. Ibáñez and H.P. Nilles, *On the low-energy $D = 4$, $N = 1$ supergravity theory extracted from the $D = 10$, $N = 1$ superstring*, *Phys. Lett.* **B 155** (1985) 65;
M. Dine, R. Rohm, N. Seiberg and E. Witten, *Gluino condensation in superstring models*, *Phys. Lett.* **B 156** (1985) 55;
T.R. Taylor, *Hidden sector of superstring models: an effective lagrangian analysis*, *Phys. Lett.* **B 164** (1985) 43.

[3] D. Lüst and T.R. Taylor, *Hidden sectors with hidden matter*, *Phys. Lett.* **B 253** (1991) 335.

[4] B. de Carlos, J.A. Casas and C. Muñoz, *Massive hidden matter and gaugino condensation*, *Phys. Lett.* **B 263** (1991) 248.

[5] B. de Carlos, J.A. Casas and C. Muñoz, *Supersymmetry breaking and determination of the unification gauge coupling constant in string theories*, *Nucl. Phys.* **B 399** (1993) 623 [hep-th/9204012].

[6] G. Veneziano and S. Yankielowicz, *An effective lagrangian for the pure $N = 1$ supersymmetric Yang-Mills theory*, *Phys. Lett.* **B 113** (1982) 231.

[7] T.R. Taylor, G. Veneziano and S. Yankielowicz, *Supersymmetric QCD and its massless limit: an effective lagrangian analysis*, *Nucl. Phys.* **B 218** (1983) 493.

[8] E. Witten, *Constraints on supersymmetry breaking*, *Nucl. Phys.* **B 202** (1982) 253.

[9] M.A. Shifman and A.I. Vainshtein, *On gluino condensation in supersymmetric gauge theories.* SU(N) *and* O(N) *groups*, *Nucl. Phys.* **B 296** (1988) 445;
D. Amati, K. Konishi, Y. Meurice, G.C. Rossi and G. Veneziano, *Nonperturbative aspects in supersymmetric gauge theories*, *Phys. Rep.* **162** (1988) 169.

[10] A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, *Supersymmetry breaking from duality invariant gaugino condensation*, *Phys. Lett.* **B 245** (1990) 401.

[11] S. Ferrara, N. Magnoli, T.R. Taylor and G. Veneziano, *Duality and supersymmetry breaking in string theory*, *Phys. Lett.* **B 245** (1990) 409.

[12] T. Banks and M. Dine, *Coping with strongly coupled string theory*, *Phys. Rev.* **D 50** (1994) 7454 [hep-th/9406132];
J.A. Casas, *The generalized dilaton supersymmetry breaking scenario*, *Phys. Lett.* **B 384** (1996) 103 [hep-th/9605180];

JHEP08(2000)013

P. Binétruy, M.K. Gaillard and Y.-Y. Wu, *Dilaton stabilization in the context of dynamical supersymmetry breaking through gaugino condensation*, *Nucl. Phys.* **B 481** (1996) 109 [hep-th/9605170];
T. Barreiro, B. de Carlos and E.J. Copeland, *On non-perturbative corrections to the kaehler potential*, *Phys. Rev.* **D 57** (1998) 7354 [hep-ph/9712443];
K. Choi, H.B. Kim and H. Kim, *Moduli stabilization in heterotic M-theory*, *Mod. Phys. Lett.* **A 14** (1999) 125 [hep-th/9808122].

[13] V.S. Kaplunovsky and J. Louis, *Model independent analysis of soft terms in effective supergravity and in string theory*, *Phys. Lett.* **B 306** (1993) 269 [hep-th/9303040];
A. Brignole, L.E. Ibáñez and C. Muñoz, *Towards a theory of soft terms for the supersymmetric standard model*, *Nucl. Phys.* **B 422** (1994) 125 [hep-ph/9308271].

[14] We give a few references from which the literature may be traced: B. de Carlos and J.M. Moreno, *Domain walls in supersymmetric QCD: from weak to strong coupling*, *Phys. Rev. Lett.* **83** (1999) 2120 [hep-th/9905165];
G. Dvali, G. Gabadadze and Z. Kakushadze, *BPS domain walls in large-N supersymmetric QCD*, *Nucl. Phys.* **B 562** (1999) 158 [hep-th/9901032];
V.S. Kaplunovsky, J. Sonnenschein and S. Yankielowicz, *Domain walls in supersymmetric Yang-Mills theories*, *Nucl. Phys.* **B 552** (1999) 209 [hep-th/9811195];
M. Shifman, *Supersymmetric domain walls (new exact results in supersymmetric gluodynamics)*, hep-th/9807166;
see also [15].

[15] A. Kovner, M. Shifman and A. Smilga, *Domain walls in supersymmetric Yang-Mills theories*, *Phys. Rev.* **D 56** (1997) 7978 [hep-th/9706089].

[16] J.A. Casas, Z. Lalak, C. Muñoz and G.G. Ross, *Hierarchical supersymmetry breaking and dynamical determination of compactification parameters by nonperturbative effects*, *Nucl. Phys.* **B 347** (1990) 243.

[17] Z. Lalak, A. Niemeyer and H.P. Nilles, *Gaugino condensation, S-duality and supersymmetry breaking in supergravity models*, *Nucl. Phys.* **B 453** (1995) 100 [hep-th/9503170].

[18] T. Dent, *Destabilizing the gaugino condensate in modular invariant supergravity models*, *Phys. Lett.* **B 470** (1999) 121 [hep-th/9909198].

[19] C.P. Burgess, J.P. Derendinger, F. Quevedo and M. Quiros, *On gaugino condensation with field-dependent gauge couplings*, *Ann. Phys. (NY)* **250** (1996) 193 [hep-th/9505171].

[20] S. Ferrara, L. Girardello, T. Kugo and A. Van Proeyen, *Relation between different auxiliary field formulations of $N = 1$ supergravity coupled to matter*, *Nucl. Phys.* **B 223** (1983) 191.

[21] L.J. Dixon, V. Kaplunovsky and J. Louis, *Moduli dependence of string loop corrections to gauge coupling constants*, *Nucl. Phys.* **B 355** (1991) 649.

[22] T.R. Taylor, *Dilaton, gaugino condensation and supersymmetry breaking*, *Phys. Lett.* **B 252** (1990) 59.

[23] M. Cvetič, A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, *Target space duality, supersymmetry breaking and the stability of classical string vacua*, *Nucl. Phys.* **B 361** (1991) 194.

[24] S.H. Shenker, *The strength of nonperturbative effects in string theory*, in *Random surfaces and quantum gravity*, Cargèse '90 Proceedings, Cargèse (France).

[25] S. Ferrara, D. Lüst, A. Shapere and S. Theisen, *Modular invariance in supersymmetric field theories*, *Phys. Lett.* **B 225** (1989) 363.

[26] S. Ferrara, . D. Lüst and S. Theisen, *Target space modular invariance and low-energy couplings in orbifold compactifications*, *Phys. Lett.* **B 233** (1989) 147.

[27] R. Rankin, *Modular forms and functions*, Cambridge University Press, Cambridge 1977.

[28] D. Lüst and C. Muñoz, *Duality invariant gaugino condensation and one loop corrected Kähler potentials in string theory*, *Phys. Lett.* **B 279** (1992) 272 [hep-th/9201047].

[29] A. Kovner and M. Shifman, *Chirally symmetric phase of supersymmetric gluodynamics*, *Phys. Rev.* **D 56** (1997) 2396 [hep-th/9702174].

[30] G. Veneziano, *Classical vs. quantum vacua: from mechanics to superstrings*, in *Vacuum and vacua: the physics of nothing*, Erice Proceedings, 1995.

[31] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, *Yang-Mills theories with local supersymmetry: lagrangian, transformation laws and superhiggs effect*, *Nucl. Phys.* **B 212** (1983) 413.

[32] P. Hořava and E. Witten, *Heterotic and type-I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209]; *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys.* **B 475** (1996) 94 [hep-th/9603142];
E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070];
for a recent review, see C. Muñoz, *Effective supergravity from heterotic M-theory and its phenomenological implications*, hep-th/9906152.

[33] L.E. Ibáñez, *The second string (phenomenology) revolution*, *Class. and Quant. Grav.* **17** (2000) 1117 [hep-ph/9911499].